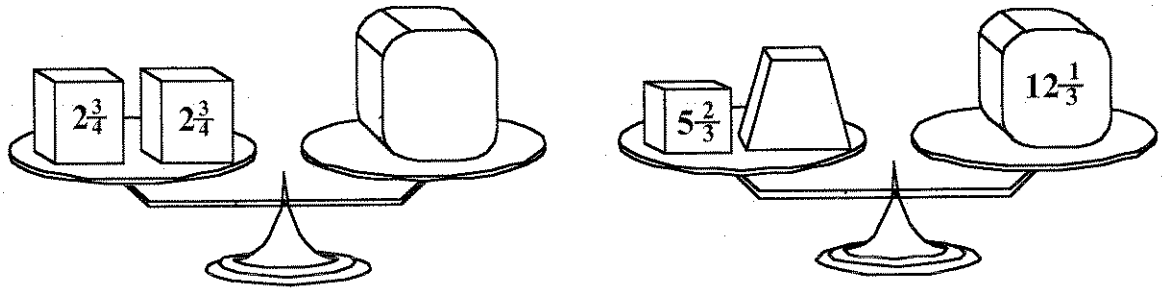
Fraction Balances

It is important for students to learn to compute with fractions in meaningful settings. If students' experience with fraction is always with abstract symbols, they may become highly procedural and fail to conceptualize addition and subtraction of fractions. Mathematics instruction is most effective when students experience ideas in a setting that is potentially meaningful to them - in a setting where they are encouraged to give meaning to their experiences rather than follow set procedures. Thus, students should experience mathematics in a variety of settings that encourage sense making.

One setting that has proven effective in helping students conceptualize addition and subtraction of fractions is a balance scale. Balancing is a bodily experience and thus natural. Thus, students can easily give meaning to tasks presented in a balance format. Unlike symbolically presented exercises, balance tasks must be interpreted and do not lend themselves to mechanical responses and thus encourage meaning making. Since no operation sign is provided, students must decide what operation to perform. It is useful to think of weights being placed on the two sides of the balance scale to make it balance.

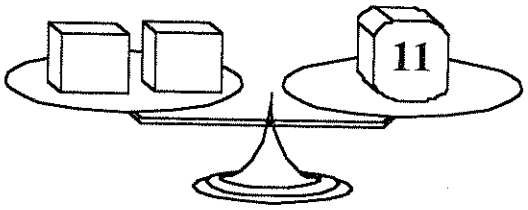
Note that a variety of arrangements are possible in putting weights on the balance. In the balance shown on the left below, the unknown is the sum of 2 3/4 and 2 3/4, which can be obtained by adding. It could be symbolized as 2 3/4 + 2 3/4 = n . The balance shown on the right requires subtraction. One could think, "What could be added to 5 2/3 that would make 12 1/3?" This can be symbolized as 5 2/3 + n = 12 1/3.



In designing the balance activities, arrangements are intentionally varied with the largest number sometimes on the right and sometimes on the left. The set of problems on a page have been carefully designed to encourage students to relate their thinking on one problem to another. Mathematics is a network of concepts, relationships and procedures.

Because we are thinking in terms of weights, it does not matter whether the two numbers on a side are interchanged, four and three is clearly the same weight as three and four. Thus the commutative property of addition becomes obvious. Students naturally construct commutitivity of addition as a relationship without any need for the term commutative.

We adopt the convention that if two boxes are identical then the same number is to go in each of them. Thus, in the figure shown below, 5 1/2 would go in each box rather than five in one box and six in the other.



This practice also builds algebraic reasoning; the balance format encourages thinking in equations. For this balance problem we could write $n + n = 11$ or $2 \times n = 11$. It is helpful to have students write number sentences for the balances they have solved. Students who solve balance tasks are "doing" algebra. The goal of the balance activities is for students to develop their own meaningful ways of thinking about adding and subtracting fractions.

Using balances with your class

After a brief discussion of Balances and perhaps a demonstration of a balance scale, present a Balance task on the overhead or chalkboard. Have students solve it and then explain their thinking. Then arrange students in pairs and give each pair one balance sheet to complete. The students working in pairs are to discuss the problems and, if possible, come to an agreement, on how to fill the boxes. This collaborative setting gives each student an opportunity to think through relationships and develop methods that make sense to him or her. Cobb has shown that learning occurs when there is disagreement in a pair and they negotiate their positions. So remember, when two students disagree there is rich potential for learning. You may want to have two or three Balance sheets available on a given day. When a pair has completed one page, you could give them a second one to consider. It is also wise to have a challenge page (more challenging balance tasks) for students who finish quickly and have developed good strategies. During this time, the goal is not to complete all the problems but to be actively engaged in thinking about the tasks - speed is not the goal.

Following the collaborative activity, bring the class together for a whole class discussion of the problems. Have students present their solution methods to the class. Providing an opportunity for students to explain and justify their reasoning is an important component of mathematics learning. Encourage students to question presenters when they do not understand or if they disagree. These whole class discussions are important for individual learning, developing an intellectual community in the classroom and providing you with much information about the students' thinking.

Fraction Balances Key

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1. 3
2. 2 1/2
3. 6 3/4
4. 6 1/4
5. 4 3/4
6. 5 1/2
7. 3 3/8
8. 3 1/3

Page 2
1. 4/5
2. 6/5 or 1 1/5
3. 6/5 or 1 1/5
4. 3/5
5. 1 2/5
6. 7 3/5
7. 6
8. 1/10

Page 3
1. 3
2. 2 1/10
3. 4 1/5
4. 4 9/10
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6. 5 3/10
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3. 4 2/3
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